

## Magnetic Dipole Moment:

$$\vec{M} = m \cdot (2\ell)$$

$m$  = pole strength,  $2\ell$  = magnetic length

**Field on Axial Line** (end-on position):

$$B_{\text{axial}} = \frac{\mu_0}{4\pi} \cdot \frac{2Mr}{(r^2 - \ell^2)^2}$$

For  $r \gg \ell$ :

$$B_{\text{axial}} \approx \frac{\mu_0}{4\pi} \cdot \frac{2M}{r^3}$$

**Field on Equatorial Line** (broad-side on):

$$B_{\text{equat}} = \frac{\mu_0}{4\pi} \cdot \frac{M}{(r^2 + \ell^2)^{3/2}}$$

For  $r \gg \ell$ :

$$B_{\text{equat}} \approx \frac{\mu_0}{4\pi} \cdot \frac{M}{r^3}$$

**Key ratio:** At equal distances,

$$B_{\text{axial}} = 2 B_{\text{equat}}$$

**Field at General Point** (angle  $\theta$  from axis):

$$B = \frac{\mu_0}{4\pi} \cdot \frac{M\sqrt{3\cos^2\theta + 1}}{r^3}$$

$$\tan \alpha = \frac{1}{2} \tan \theta \quad (\alpha = \text{angle of } B \text{ with } r)$$

## 2. Torque, PE and Oscillations

**Torque on dipole in uniform field:**

$$\vec{\tau} = \vec{M} \times \vec{B} = MB \sin \theta$$

**Potential energy:**

$$U = -\vec{M} \cdot \vec{B} = -MB \cos \theta$$

- Equilibrium (stable):  $\theta = 0^\circ$ ,  $U = -MB$
- Equilibrium (unstable):  $\theta = 180^\circ$ ,  $U = +MB$

**Work done to rotate from  $\theta_1$  to  $\theta_2$ :**

$$W = MB(\cos \theta_1 - \cos \theta_2)$$

**Time period of oscillation in field  $B$ :**

$$T = 2\pi \sqrt{\frac{I}{MB}}$$

$I$  = moment of inertia of magnet

**Frequency:**

$$f = \frac{1}{2\pi} \sqrt{\frac{MB}{I}}$$

**Magnetic moment of current loop:**

$$M = NIA$$

$N$  = turns,  $I$  = current,  $A$  = area

**For circular loop of radius  $r$ :**

$$M = NI\pi r^2$$

**Gyromagnetic ratio (electron):**

$$\frac{M_l}{L} = \frac{e}{2m_e} = 8.8 \times 10^{10} \text{ C kg}^{-1}$$

**Bohr Magnetron:**

$$\mu_B = \frac{eh}{4\pi m_e} = 9.27 \times 10^{-24} \text{ J T}^{-1}$$

$$\oint \vec{B} \cdot d\vec{A} = 0$$

- Magnetic monopoles **do not exist**
- Magnetic field lines are **closed loops**
- Contrast: Gauss's law for  $E$  gives  $\frac{q}{\epsilon_0}$

## 5. Earth's Magnetism

**Three elements of Earth's magnetism:**

Element	Definition
Declination ( $\delta$ )	Angle between geographic north and magnetic north
Dip / Inclination ( $\phi$ )	Angle between $B_E$ and horizontal plane
Horizontal component	$H = B_E \cos \phi$
Vertical component	$V = B_E \sin \phi$

**Key relations:**

$$V = H \tan \phi \quad B_E = \sqrt{H^2 + V^2}$$

$$\tan \phi = \frac{V}{H}$$

**Apparent dip** (when vertical plane turned by angle  $\alpha$ ):

$$\tan \phi' = \frac{\tan \phi}{\cos \alpha}$$

**At magnetic poles:**  $\phi = 90^\circ$ ,  $H = 0$

**At magnetic equator:**  $\phi = 0^\circ$ ,  $V = 0$

- Fe:  $\approx 1043$  K
- Ni:  $\approx 631$  K
- Co:  $\approx 1388$  K

**Effect of temperature on diamagnetics:** Nearly independent of temperature.

**Magnetisation:**

$$\vec{M} = \frac{\text{Net magnetic moment}}{\text{Volume}} \quad [\text{A m}^{-1}]$$

**Magnetic intensity / field strength:**

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M} \Rightarrow \vec{B} = \mu_0(\vec{H} + \vec{M})$$

**Magnetic susceptibility:**

$$\chi_m = \frac{M}{H}$$

**Relative permeability:**

$$\mu_r = 1 + \chi_m \quad \mu = \mu_0 \mu_r$$

**Relation:**

$$\vec{B} = \mu_0 \mu_r \vec{H}$$

1.  $B_{\text{axial}} = \frac{\mu_0}{4\pi} \cdot \frac{2M}{r^3}$  (most tested)
2.  $B_{\text{equat}} = \frac{\mu_0}{4\pi} \cdot \frac{M}{r^3}$  and  $B_A = 2B_E$
3.  $T = 2\pi \sqrt{I/MB}$  — compare two magnets!
4.  $V = H \tan \phi$ ,  $B_E = \sqrt{H^2 + V^2}$
5.  $\chi_m = C/T$  and  $\mu_r = 1 + \chi_m$
6.  $\tau = MB \sin \theta$ ,  $U = -MB \cos \theta$
7.  $M = NIA$  (current loop analogy)

- **Axial vs Equatorial:** Axial field is *along* the axis (same direction as  $M$ ); equatorial is *opposite* to  $M$ . Students often forget the direction reversal.
- **Magnetic length  $\neq$  geometric length:** Effective magnetic length =  $0.84 \times$  geometric length for a bar magnet.
- **$H$  vs  $B$ :**  $H$  is magnetic field intensity (A/m);  $B$  is magnetic flux density (T). They differ inside a medium.  $B = \mu_0 H$  only in vacuum.
- **Dip  $\neq$  Declination:** Dip is measured in the vertical plane; declination is in the horizontal plane.
- **Curie vs Curie-Weiss:** Curie law applies to paramagnetics; Curie-Weiss law applies to ferromagnetics above  $T_C$ .
- **Diamagnetic susceptibility:**  $\chi_m$  is negative (repelled), NOT zero. Students confuse it with vacuum ( $\chi_m = 0$ ).
- $\mu_r < 1$  for diamagnetics: Since  $\chi_m$  is small and negative,  $\mu_r = 1 + \chi_m$  is slightly less than 1.
- **Gauss's law for magnetism:** Surface integral of  $B$  is zero; this does *not* mean  $B = 0$  everywhere on the surface.
- **Apparent dip:**  $\tan \phi' = \tan \phi / \cos \alpha$ , NOT  $\tan \phi \cdot \cos \alpha$ .
- **Time period comparison:** When two magnets are placed N-N or N-S, use  $M_{\text{eff}} = M_1 + M_2$  or  $M_1 - M_2$  in  $T$  formula.

Property	Dia	Para
$\chi_m$	Small, -ve	Small, +ve
$\mu_r$	$< 1$	$> 1$ (slightly)
Behaviour in field	Weakly repelled	Weakly attracted
Examples	Cu, Bi, Au, Water	Al, Na, O <sub>2</sub>

**Ferromagnetic materials:** Fe, Co, Ni  
 $\chi_m \gg 1$ ,  $\mu_r \gg 1$ , strongly attracted.

## 8. Curie's Law & Curie-Weiss Law

**Curie's Law** (paramagnetic):

$$\chi_m = \frac{C}{T}$$

$C$  = Curie constant,  $T$  = absolute temperature

**Curie-Weiss Law** (ferromagnetic above  $T_C$ ):

$$\chi_m = \frac{C}{T - T_C}$$

$T_C$  = Curie temperature (ferromagnet becomes paramagnetic above  $T_C$ )

**Key Curie temperatures:**

## 9. Hysteresis & Permanent Magnets

**Hysteresis Loop terms:**

- **Retentivity** (Remanence): Value of  $B$  when  $H = 0$  (residual magnetism)
- **Coercivity:** Value of  $H$  needed to demagnetise

( $B = 0$ )

- **Hysteresis loss**  $\propto$  area of B-H loop

Property	Perm. Mag-	Electromagnet
Retentivity	High	High
Coercivity	High	Low
Material	Steel, Alnico	Soft iron
Hysteresis area	Large	Small

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$$

$$\frac{\mu_0}{4\pi} = 10^{-7} \text{ T m A}^{-1}$$

$$\mu_B = 9.27 \times 10^{-24} \text{ J T}^{-1}$$

$$B_E \text{ (Earth)} \approx 4 \times 10^{-5} \text{ T (at equator)}$$

Prepared by: Let's Play With Physics • [letsplaywithphysics.in](https://letsplaywithphysics.in) • *Master the concept, not just the formula.*

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